How to discover charm CP violation

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CP violation in $D$ decays

I discuss hadronic two-body weak decays of $D^+, D^0, D_s^+$ mesons.

$$D^+ \sim c\bar{d}, \quad D^0 \sim c\bar{u}, \quad D_s^+ \sim c\bar{s},$$

Examples: $D^+ \rightarrow \bar{K}^0\pi^+$, $D^0 \rightarrow \pi^+\pi^-$, $D^+ \rightarrow K^0\pi^+$.

Decays are classified in terms of powers of the \textit{Wolfenstein parameter}

$$\lambda \sim |V_{us}| \sim |V_{cd}| \sim 0.22.$$

Amplitude $A \propto \begin{cases} 
\lambda^0 & \text{Cabibbo-favoured} \\
\lambda^1 & \text{singly Cabibbo-suppressed} \\
\lambda^2 & \text{doubly Cabibbo-suppressed}
\end{cases}$
Operators from $W$ exchange, e.g.

$$Q_{1}^{sd} = \bar{s}_{L}^{i} \gamma_{\mu} c_{L}^{k} \bar{u}_{L}^{k} \gamma^{\mu} d_{L}^{j}$$

$$Q_{2}^{sd} = \bar{s}_{L}^{i} \gamma_{\mu} c_{L}^{j} \bar{u}_{L}^{k} \gamma^{\mu} d_{L}^{k}$$

with colour indices $j, k$. 
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with colour indices $j, k$.

Strip CKM factors off the amplitudes:

$$\mathcal{A}^{CF} \equiv V^{\ast}_{cs} V_{ud} A$$

$$\mathcal{A}^{DCS} \equiv V^{\ast}_{cd} V_{us} A.$$
In the SCS amplitudes three CKM structures appear:

\[ \lambda_d = V_{cd}^* V_{ud}, \quad \lambda_s = V_{cs}^* V_{us}, \quad \lambda_b = V_{cb}^* V_{ub} \]

and CKM unitarity \( \lambda_d + \lambda_s + \lambda_b = 0 \) is invoked to eliminate one of these.

Commonly used

\[ A^{SCS} \equiv \lambda_{sd} A_{sd} - \frac{\lambda_b}{2} A_b \]

with

\[ \lambda_{sd} = \frac{\lambda_s - \lambda_d}{2} \quad \text{and} \quad -\frac{\lambda_b}{2} = \frac{\lambda_s + \lambda_d}{2} \]
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In view of \( |\lambda_b|/|\lambda_{sd}| \sim 10^{-3} \) only \( A_{sd} \) is relevant for branching ratios.

Penguin loop contributions to \( A_{sd} \) are GIM-suppressed (naively: \( \propto (m_s^2 - m_d^2)/m_c^2 \)).
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... are not discovered yet!
Direct CP asymmetries in singly Cabibbo-suppressed decays: 
With $A^{SCS} = A$ write

$$A = \lambda_{sd} A_{sd} - \frac{\lambda_b}{2} A_b,$$

CP-conjugate decay: $$\overline{A} = -\lambda_{sd}^* A_{sd} + \frac{\lambda_b^*}{2} A_b.$$ 

Find

$$a_{CP}^{dir} \equiv \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} = \text{Im} \frac{\lambda_b}{\lambda_{sd}} \text{Im} \frac{A_b}{A_{sd}}.$$ 

Recall: $|A_{sd}| = |A|/|\lambda_{sd}|$ is fixed from measured branching ratios.

$\Rightarrow$ need $|A_b|$ and the phase of $A_b/A_{sd}$ to predict $a_{CP}^{dir}$. 

Ulrich Nierste (TTP)
All SM predictions for CP asymmetries involve a suppression by \( \text{Im} \frac{\lambda_b}{\lambda_{sd}} = -6 \cdot 10^{-4} \). This is also true for mixing-induced CP asymmetries or the semileptonic CP asymmetry, which quantifies CP violation (CPV) in mixing.
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In the pre-LHC era CPV could have only been discovered if there was a substantial enhancement by new physics, with $\text{Im} \frac{\lambda_b}{\lambda_{sd}}$ replaced by some $\mathcal{O}(1)$ factor. Thus the “CPV discovery channels” were identical to the “new-physics discovery channels”.

With LHCb probing CP asymmetries down to SM predictions, the goals

(a) “discover CPV if there is no physics beyond the SM”
and

(b) “discover new physics”

require different strategies:

For (a) need decay modes with large SM predictions for $a_{CP}^{dir}$.
For (b) need decay modes with clean SM predictions for $a_{CP}^{dir}$.
Example: mixing-induced CPV

If there is new physics in charm FCNC transitions driven by particles with mass $M \gg M_W$, it will (by default) have a much larger imprint on $D - \bar{D}$ mixing than on hadronic charm decay amplitudes.

⇒ use superweak approximation, i.e. neglect CPV in decay amplitudes
Example: mixing-induced CPV

If there is new physics in charm FCNC transitions driven by particles with mass $M \gg M_W$, it will (by default) have a much larger imprint on $D - \bar{D}$ mixing than on hadronic charm decay amplitudes.

$\Rightarrow$ use superweak approximation, i.e. neglect CPV in decay amplitudes

If your goal is CPV discovery within the SM, the superweak approximation is inconsistent, because you keep $\text{Im} \frac{\lambda_b}{\lambda_{sd}}$ in the $D - \bar{D}$ mixing amplitude while neglecting it elsewhere. Moreover, depending on the decay channel, direct CPV can be much larger than mixing-induced CPV.

$\Rightarrow$ Don’t use the superweak approximation!
CPV discovery channels in the SM

\[ a_{CP}^{dir} = \text{Im} \left( \frac{\lambda_b}{\lambda_{sd}} \right) \text{Im} \left( \frac{A_b}{A_{sd}} \right) \]

\[ = -6 \cdot 10^{-4} \text{Im} \left( \frac{A_b}{A_{sd}} \right) \]

can be $O(10)$ in the SM, if $A_{sd}$ is suppressed.

Typical SM values of $a_{CP}^{dir}$ are below $10^{-3}$, thus identifying decays with large $\left| \frac{A_b}{A_{sd}} \right|$ is important. (The phase $\text{arg} \left( \frac{A_b}{A_{sd}} \right)$ is unpredictable, so one must be lucky.)
Use the approximate SU(3)$_F$ symmetry of QCD: Owing to $m_{u,d,s} \ll \Lambda_{\text{QCD}}$ hadronic amplitudes are approximately invariant under unitary rotations of

$$
\begin{pmatrix}
  u \\
  d \\
  s
\end{pmatrix}.
$$

$\Rightarrow$ One can correlate various $D \to K\pi$ decays.

Example: In the limit of exact SU(3)$_F$ symmetry find

$$
\mathcal{B}(D^0 \to \pi^+\pi^-) = \mathcal{B}(D^0 \to K^+K^-).
$$

Data show $\mathcal{O}(30\%)$ SU(3)$_F$ breaking in the decay amplitudes. It is possible to include SU(3)$_F$ breaking to first order (linear breaking) in the decomposition of the decay amplitudes in terms of SU(3)$_F$ representations.

$SU(3)_F$ limit:

- tree (T)
- color-suppressed tree (C)
- exchange (E)
- annihilation (A)
SU(3)$_F$ breaking

Feynman rule from $H_{SU(3)_F} = (m_s - m_d)\bar{s}s$: dot on $s$-quark line. Find 14 new topological amplitudes such as

$T_1$  $T_2$  ...

Important:

penguin ($P_{\text{break}}$)
Predict CP asymmetries in $D$ decays

The theory community has delivered a **perfect service** to the experimental colleagues:
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- within the Standard Model
  and
- as evidence for new physics!
CP asymmetries

Generic problem: For CP asymmetries we need $A_b$ which involves new hadronic quantities which do not appear in $A_{sd}$ and are therefore not constrained by branching fractions.

E.g. new SU(3) representations or, in our analysis, new topological-amplitudes.

Prominent example:

Penguins $P_s$ and $P_d$ appear in other combinations than $P_{\text{break}} = P_s - P_d$. We also need $P \equiv P_s + P_d - 2P_b$. 
Experimentally $a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-)$ and $a_{CP}^{\text{dir}}(D^0 \to K^+K^-)$ are well constrained. Status of 2015:

$\Delta a_{CP}^{\text{dir}} \equiv a_{CP}^{\text{dir}}(D^0 \to K^+K^-) - a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-) = -0.00253 \pm 0.00104$

$\Sigma a_{CP}^{\text{dir}} \equiv a_{CP}^{\text{dir}}(D^0 \to K^+K^-) + a_{CP}^{\text{dir}}(D^0 \to \pi^+\pi^-) = -0.0011 \pm 0.0026$

Topological amplitudes:

$A_{sd}(D^0 \to \pi^+\pi^-) = -T - E + P_{\text{break}}$

$A_{b}(D^0 \to \pi^+\pi^-) = T + E + P + PA$

Penguin annihilation diagram $PA$: 

![Penguin annihilation diagram](image-url)
It is useful to eliminate $T + E$ in $A_b$ in favour of $A_{sd}$:

$$A_b(D^0 \to \pi^+\pi^-) = -A_{sd}(D^0 \to \pi^+\pi^-) + P_{\text{break}} + P + PA$$

$$\Rightarrow \quad \text{Im} \frac{A_b(\pi^+\pi^-)}{A_{sd}(\pi^+\pi^-)} = \text{Im} \frac{P_{\text{break}} + P + PA}{A_{sd}(\pi^+\pi^-)}$$

Similarly for $D^0 \to K^+K^-$ (up to SU(3)$_F$ breaking):

$$\text{Im} \frac{A_b(K^+K^-)}{A_{sd}(K^+K^-)} = \text{Im} \frac{P_{\text{break}} - P - PA}{A_{sd}(\pi^+\pi^-)}$$

Thus $\Delta a^\text{dir}_{CP}$ rules out spectacular enhancements of $P + PA$ and $\Sigma a^\text{dir}_{CP}$ likewise constrains $P_{\text{break}}$.

$$\Rightarrow \quad \text{To find CPV look for alternatives to } P, PA!$$
\[ \mathcal{A}(D^0 \to K_S K_S) = \lambda_{sd} A_{sd} - \frac{\lambda_b}{2} A_b. \]

Special feature I:

In the SU(3)_F limit: \( A_{sd} = 0 \) while \( A_b \neq 0 \)

\[ \Rightarrow \text{suppressed } \mathcal{B}(D^0 \to K_S K_S) = (1.7 \pm 0.4) \cdot 10^{-4} \]

enhanced \( a_{CP}^{dir} \propto \text{Im} \frac{A_b}{A_{sd}} \)
Special feature II:

\[ a^{\text{dir}}_{CP}(D^0 \rightarrow K_S K_S) \] receives contributions at tree level, from the (sizable!) exchange diagram:
Result: $a_{CP}^{\text{dir}}$ can be large. We find:

$$|a_{CP}^{\text{dir}}(D^0 \to K_SK_S)| \leq 1.1\% \quad \text{@95\% C.L.}$$

The CP violation in $K^-K^+$ mixing is meant to be subtracted.

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Experiment determines

$$A_{CP} = a_{CP}^{\text{dir}} - A_{\Gamma} \frac{\langle t \rangle}{\tau},$$

where $\langle t \rangle$ is the average decay time and $\tau$ is the $D^0$ lifetime.

$$A_{CP}^{\text{CLEO 2001}} = -0.23 \pm 0.19$$

$$A_{CP}^{\text{LHCb 2015}} = -0.029 \pm 0.052 \pm 0.022$$

$$A_{CP}^{\text{Belle 2016}} = -0.0002 \pm 0.0153 \pm 0.0017$$
Two $D^0 \rightarrow KK^*$ decays:

$$D^0 \rightarrow K^*0 \rightarrow K^-\pi^+K^0$$

$$D^0 \rightarrow K^*0 \rightarrow K^+\pi^-K^0$$

with the $K^0$, $\bar{K}^0$ hadronising into $K_S$.

Write shortly:

$$A(K^*0) \equiv A(D^0 \rightarrow K^*0K^0)$$

$$A(K^*0) \equiv A(D^0 \rightarrow K^*0\bar{K}^0).$$
Each diagram comes in two variants, e.g.

\[ D^0 \rightarrow KK^* \]

- \( E_P \) means “pseudoscalar”, \( V \) means “vector”.
Topological amplitudes:

\[ A_{sd}(K^*0) = E_P - E_V + E_{P3} - E_{V1} - E_{V2} - P_{A_{PV}}^{\text{break}} \]

\[ A_{b}(K^*0) = -E_P - E_V - E_{P3} - E_{V1} - E_{V2} - P_{A_{PV}} \]

\[ = A_{sd}(K^*0) - 2E_P - 2E_{P3} - P_{A_{PV}} + P_{A_{PV}}^{\text{break}} \]

\[ A_{sd}(\overline{K}^*0) = -E_P + E_V - E_{P1} - E_{P2} + E_{V3} - P_{A_{PV}}^{\text{break}} \]

\[ A_{b}(\overline{K}^*0) = -E_P - E_V - E_{P1} - E_{P2} - E_{V3} - P_{A_{PV}} \]

\[ = A_{sd}(\overline{K}^*0) - 2E_V - 2E_{V3} - P_{A_{PV}} + P_{A_{PV}}^{\text{break}} \]

\[ \Rightarrow \quad a_{CP}^{\text{dir}}(D^0 \rightarrow \overline{K}^*0 K^0) = \text{Im} \frac{\lambda_b}{\lambda_{sd}} \frac{A_{b}(\overline{K}^*0)}{A_{sd}(K^*0)} \]

\[ \approx -\text{Im} \frac{\lambda_b}{\lambda_{sd}} \frac{A_{b}(K^*0)}{A_{sd}(K^*0)} = -a_{CP}^{\text{dir}}(D^0 \rightarrow K^*0 \overline{K}^0) = a_{CP}^{\text{dir}}(\overline{D}^0 \rightarrow \overline{K}^*0 K^0) \]
\( a_{CP}^{\text{dir}}(D^0 \to \bar{K}^0 K^0) \approx a_{CP}^{\text{dir}}(\bar{D}^0 \to \bar{K}^0 K^0) \) means that no flavour tagging is needed:

\[
a_{CP}^{\text{dir}}(\bar{D} \to K_S K^{0*}) \approx a_{CP}^{\text{dir}}(D^0 \to K_S K^{0*})
\]

Using

\[
\mathcal{B}^{\text{exp}}(D^0 \to K^{*0} K_S) = (1.1 \pm 0.2) \cdot 10^{-4},
\]

\[
\mathcal{B}^{\text{exp}}(D^0 \to \bar{K}^{*0} K_S) = (0.9 \pm 0.2) \cdot 10^{-4}.
\]

from experiment to determine \( |E_P - E_V| = (1.6 \pm 0.2) \cdot 10^{-6} \) we find

\[
|a_{CP}^{\text{dir, untag}}| \lesssim 0.003.
\]

The maximum \( |a_{CP}^{\text{dir, untag}}| = 0.003 \) corresponds to \( \arg(E_V/E_P) = 0.14 \pi \).

Another goodie: One can scan the \( K^+ \pi^- K_S \) Dalitz plot near the \( K^{*0} \) resonance for a favourable \( \arg(E_V/E_P) \).
“Charm CPV discovery within the SM” and “New-physics discovery through CPV” require different strategies.

Within the Standard Model the direct CP asymmetry in the charm decay in $D^0 \rightarrow K_SK_S$ can be as large as 1.1%. $a_{CP}^{dir}(D^0 \rightarrow K_SK_S)$ is dominated by the exchange diagram, which involves no loop suppression. View $D^0 \rightarrow K_SK_S$ as a discovery channel for charm CP violation.

The same is true for $D^0 \rightarrow K^{*0}K_S$, which moreover requires no tagging to measure $a_{CP}^{dir}$. $a_{CP}^{dir, untag}(D^0 \rightarrow K^{*0}K_S)$ can be as large as 0.3%. I argue that $D^0 \rightarrow K^{*0}K_S$ is the best discovery channel for charm CP violation.
Backup
Number of $D^+, D^0, D_s^+$ decay modes:

4 Cabibbo-favoured,
5 doubly Cabibbo-suppressed,
8 singly Cabibbo-suppressed.

Cabibbo-favoured (CF), $A \propto \lambda^0$
doubly Cabibbo-suppressed (DCS), $A \propto \lambda^2$
singly Cabibbo-suppressed (SCS), $A \propto \lambda^1$
To learn as much as possible about $A_{sd}$ for the various decay modes, do a correlated analysis of all available data on the branching fractions of

$D^0 \rightarrow K^+K^-$, $D^0 \rightarrow \pi^+\pi^-$, $D^0 \rightarrow K_SK_S$, $D^0 \rightarrow \pi^0\pi^0$, $D^+ \rightarrow \pi^0\pi^+$, $D^+ \rightarrow K_SK^+$, $D_s^+ \rightarrow K_S\pi^+$, $D_s^+ \rightarrow K^+\pi^0$, $D^0 \rightarrow K^-\pi^+$, $D^0 \rightarrow K_S\pi^0$, $D^0 \rightarrow K_L\pi^0$, $D^+ \rightarrow K_S\pi^+$, $D^+ \rightarrow K_L\pi^+$, $D_s^+ \rightarrow K_SK^+$, $D^0 \rightarrow K^+\pi^-$, $D^+ \rightarrow K^+\pi^0$,

and the $K^+\pi^-$ strong phase difference $\delta_{K\pi} = 6.45^\circ \pm 10.65^\circ$.

This gives essentially one ingredient of the CP asymmetries, $|A_{sd}|$, but gives no information on $|A_b|$ and arg$(A_b/A_{sd})$.

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